Calculation of BLDC Motor Parameters:

In order to model a BLDC motor with good accuracy, the armature self inductance per phase, the armature resistance, the moment of inertia of the rotor, and the viscous friction coefficient must be known. The back emf constant is already known as it is the inverse of kv which is given and the torque constant is also known as it is equal to kv, a necessary result of the conservation of mechanical and electrical energy. The armature mutual inductances per phase, eddy and hysteresis losses, as well as the variation in inductance with respect to rotor phase angle are beyond what is necessary for the given project goals. As such the following test plan to materialize the motor(s) will be implemented upon acquisition.

1. Armature resistance will be determined via usage of an ohmmeter applied to both ends of the Armature.

2.

To solve for the armature inductance (L), a small AC voltage with frequency ω will be created via a multifunction generator and be applied to the armature, the current and voltage will then be measured using an ohmmeter. From there:

$$Impedence(Z) = \sqrt{reactance^2 + resistence^2}$$

Resistance in this case is the armature Resistance measured in the previous part. Rearranging to solve for the reactance gets:

$$reactance = \sqrt{Impedence^2 - resistence^2} = 2\pi\omega L \rightarrow L = \frac{\sqrt{Impedence^2 - Resistence^2}}{(2 * pi * \omega)}$$

The moment of inertia for the rotor can be found via suspending the motor on a pendulum made up of two wires by its rotor, as shown in the diagram below. The formulas at the bottom of the image are:

$$J = \frac{m * g * d^2}{4\pi^2 * f^2 * l}$$

$$f = \frac{1}{T}$$

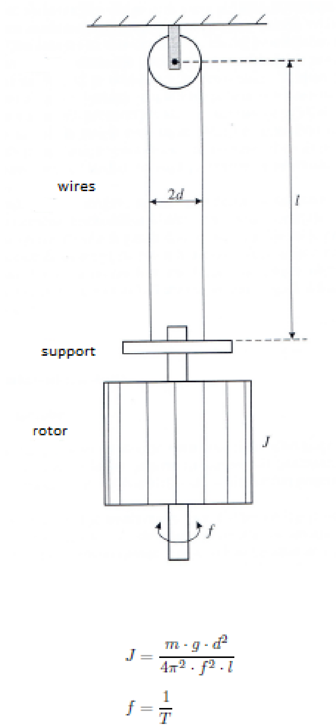

Figure 1: Setup for measuring rotor inertia

4. The equations for a BLDC motor, keeping the simplifying assumptions previously made is:

$$\frac{di_a}{dt} = -R(L-M) * I_a + \frac{E_a}{J} + \frac{1}{(L-M)) * V_a};$$
$$\frac{di_b}{dt} = -R(L-M) * I_b + \frac{E_b}{J} + \frac{1}{(L-M)) * V_b};$$
$$\frac{di_c}{dt} = -R(L-M) * I_c + \frac{E_c}{J} + \frac{1}{(L-M)) * V_c};$$
$$\frac{di_b}{dt} = -R(L-M) * I_b + \frac{E_b}{J} + 1(L-M)) * V_b;$$
$$\frac{di_c}{dt} = -R(L-M) * I_c + \frac{E_b}{J} + \frac{1}{(L-M))} * V_c;$$
$$\frac{\omega}{dt} = \frac{(E_a * I_a + E_b * I_b + E_c * I_c)}{J} - B_v * \frac{\Omega}{J} - \frac{T_{load}}{J};$$
$$\frac{\Theta}{dt} = P * \frac{Omega}{2};$$

where Back-EMF's:
$E_a = \Lambda * F_a * \Omega;$
$E_b = \Lambda * F_b * \Omega;$

$E_c = \Lambda * F_c * \Omega;$

I_a, I_b, I_c= phase currents
V_a, V_b, V_c= phase voltages (trapezoidal)
Λ = total PM flux linkage;
B_v = Viscous Friction constant;
ω = rotor angular speed;
θ = rotor position;
F_a, F_b, F_c = trapezoidal function for the phases i=a, b, c
$f_b(\theta) = f_a(\theta + 120)$; $f_c(\theta) = f_a(\theta + 240)$
J = Rotor Moment of Inertia;
R = Windings Resistance;
L = Windings self Inductance;
M = Windings Mutual Inductance;
T_load = Torque Load;
P = Number of poles;

The basic truth of the model is that current is directly proportional to torque. Moreover, torque is inversely proportional to rotational speed. The reason is, is that when the motor starts spinning, the full voltage is dropping across the armature, which means current and thus torque is maximized at startup. However, as motor speed increases, the changing magnetic field of the rotor induces a voltage in the stator that is directly opposed to the battery voltage as per faraday's law. This causes the voltage difference across the armature, and hence current and so torque to decrease. The faster the motor spins, the faster the magnetic field changes and the larger the induced voltage on the stator. At a certain RPM, if the motor is ideal and unloaded, which is the motor's top speed, this induced voltage (called the back EMF) becomes equal to voltage being applied to the armature. At that point the armature current is zero and so the torque is zero.

Keeping this in mind, the derivation of the value of viscous friction coeffient is relatively easy. One merely brings the motor up to full speed with no load and then measures the current flowing through the armature phases. Those 3 phase currents can then be used in a rearranged torque equation to solve for the viscous friction coefficient:

$$\frac{(Ea * I_a + E_b * I_b + E_c * I_c}{\Omega} = B_v$$